Last update: 2:53am (Yerevan), May 13.

Mathematical Logic HOMEWORK 10 **Due: May 17 (Fri)**

1. Let $\boldsymbol{N} := (\mathbb{N}, 0, S, +, \cdot)$ be the standard structure of natural numbers, and prove:

Theorem (Tarski). *The set* $\ulcorner \mathrm{Th}(\boldsymbol{N}) \urcorner$ *of codes of the theory* $\mathrm{Th}(\boldsymbol{N})$ *is not arithmetical.*

HINT: Use the fixed point lemma.

2. Let $R_1, \ldots, R_m \subseteq \mathbb{N}^k$ be computable relations such that for each $\vec{a} \in \mathbb{N}^k$ exactly one of $R_1(\vec{a}), \ldots, R_m(\vec{a})$ holds, and suppose that $g_1, \ldots, g_m : \mathbb{N}^k \to \mathbb{N}$ are computable functions. Then $g : \mathbb{N}^k \to \mathbb{N}$ given by

$$g(\vec{a}) := \begin{cases} g_1(\vec{a}) & \text{if } R_1(\vec{a}) \\ \vdots & \vdots \\ g_m(\vec{a}) & \text{if } R_m(\vec{a}) \end{cases}$$

is computable.

3. Prove that the following functions are computable.

(a) $\dot{-} : \mathbb{N}^2 \to \mathbb{N}$ defined by $n \dot{-} m := \begin{cases} n - m & \text{if } n \geqslant m \\ 0 & \text{otherwise.} \end{cases}$

(b) $\mathrm{Rem} : \mathbb{N}^2 \to \mathbb{N}$ defined by $\mathrm{Rem}(n, m) :=$ the unique $r \in \{0, 1, \ldots m-1\}$ such that $n = q \cdot m + r$ if $m \neq 0$; otherwise, $\mathrm{Rem}(n, m) := 0$.

4. It is an open question as to whether the decimal representation 3.1415926... of the number π contains arbitrarily large strings of consecutive 0s. Nevertheless, prove that the following function $f : \mathbb{N} \to \mathbb{N}$ is computable:

$$f(n) := \begin{cases} 1 & \text{if the decimal representation of } \pi \text{ contains } n \text{ consecutive 0s} \\ 0 & \text{otherwise.} \end{cases}$$