Mathematical Logic HOMEWORK 9 **Due: May 1 (Wed)**

1. Call a collection $\mathcal{S}$ of sets **nested** if $A \subseteq B$ or $B \subseteq A$ for any two sets $A, B \in \mathcal{S}$. Prove that for a nested collection $\mathcal{S}$ of consistent σ-theories, the union $T_\infty := \bigcup \mathcal{T} := \bigcup_{T \in \mathcal{S}} T$ is consistent.

2. Let σ be a signature and prove:

   (a) $\vdash (t = t)$ for each σ-term t.

   REMARK: I said during lecture that there would be an issue with this because of my convention that a quantified variable does not appear outside of the range of quantification, but I don't see the issue anymore. Please let me know if there is an issue after all.

   (b) $\vdash (\varphi(t/v) \to \exists v \varphi)$ for each σ-formula φ and each σ-term t that is okay to plug in for v in φ and that does not contain the variable v.

   (c) $\vdash \exists v (t = v)$ for each σ-term t that does not contain the variable v.

3. Prove from scratch the following **special case of Gödel's Completeness theorem**: Let σ be a finite signature and let $\varphi := \exists v_1 \dots \exists v_5 (\psi \wedge \forall v \bigvee_{i=1}^{5} v = v_i)$, where $\psi(v_1, \dots, v_5)$ is a quantifier free extended σ-formula.