Math Logic: Model Theory & Computability Lecture 10

Thus, it is desicable to come up with a hopefully equivalent theory to Th (N) those axions would easily recognizable (say, by a computer project). Such an attempt was made by Peano, who suspected the following theory, now called Peano Acilhuetic (PA), in the signchare Jark = (D, S, +, .): $(PA1) \forall x (0 \neq S(x))$ [D is not in the incyclof S] $(PA 2) \forall x \forall y (S(x) = S(y) \rightarrow x = y) [S is injective]$ $(PA 3) \forall x (x + 0 = x) [D is the additive identity]$ $(PA 4) \forall x \forall y (x + S(y) = S(x + y)) [det. of + via S]$ $(PA 5) \forall x (x \cdot 0 = 0) [0 is the multiplicative annihilator]$ $(PA6) \forall x \forall y (x \cdot S(y) = x \cdot y + x) \quad [def. of ~ via +]$ (PA7^{oo}) The axiom schema of induction: for each extended Japp-formula Y(x, j) the tollovicy is an axiom of PA: $\forall \vec{y} \left[\Psi(0, \vec{y}) \land \forall x \left(\Psi(x, \vec{y}) \rightarrow \Psi(S(x), \vec{y}) \right) \rightarrow \forall x \ \Psi(x, \vec{y}) \right],$ Mu Vig abbreviates Vy, Vy, Mene Z = (5, ,..., yk). Penno hoped but PA would be an equivalent theory to Th(N), but bodded proved that this is not the case, in fact, there is no computer recognizable theory equivalent to Th(N) - this is known as the bodded incompleteness Theorem. Semantic consistency, in plication, and completeness. Det. A J-Heory is called satisfiable (semantically consistent) if it has a utilizing hyperbolic us hacknessing

To prove this, cycin tix any field at charteristic p and show ht the statement holds in it. (c) FJELDS, = 1+1+...+1=0 for all n e IN⁴. To prove this fix an arbitrary field of char. O and show this by incluction. Def. a-strachoes A and B are called elementarily equivalent if they have the same theory, i.e. Th (A) = Th (B). We denote this by A = B. We have proven earlier that it A and B one isomorphic, the key one elementarity equivalent. However the converse isn't true in general. For exam-ple, one can show (HW) that (OR, c) = (R, c) but they can't be iso-morphic bend OR and IR are not equivanerous. Det. let T be a s-theory. We say that T is semachically s-couplete if for each s-section &, we have that T + & or T = - P. Note It T is not subistiable, then T is automatically couplete hence both TF & and TF-V for each J-sendence &. Thus, this notion is only useful when T is satisfiable, in which case the "or" is exclusive, i.e. only one of TFP and TF-Y helds. Prop. A J-theory T is semachically J-complete it and only if A=B hor all models A, B of T. Exceptes. (a) GROUPS is not sumanifiedly couplete bense, for excepte, there are